

Small-time local controllability of a class of control systems

Mikhail Krastanov^{1,2} and Margarita Nikolova²

¹Bulgarian Academy of Science, Institute of Mathematics and Informatics, Sofia, Bulgaria

²Sofia University, Sofia, Bulgaria

Abstract

Small-time local controllability (STLC) at a point x_0 is a fundamental property of control systems, intimately connected to the local structure of their reachable sets. This study builds upon the notion of a *tangent vector field* to the reachable set of a control system (a concept introduced by Hermes in [2], based on an idea of Krener (cf. [6]). The importance of this concept stems from the fact that the set $E^+(x_0)$, consisting of all tangent vector fields to the reachable set at x_0 , forms a convex cone. If the zero vector lies in the interior of this cone, the system is STLC at x_0 . A long-standing open question concerns the precise characterization of the set $E^+(x_0)$.

A class of control-affine systems with bounded controls is considered and the Lie algebra generated by the corresponding vector fields is studied. By applying the classical Campbell–Baker–Hausdorff formula from Lie group theory and using the exponential representation of flows and the chronological calculus, new elements of the set $E^+(x_0)$ are derived. Sufficient conditions, necessary conditions and the gap between them is discussed. Examples illustrate the obtained results.

References

- [1] A. Agrachev, R. Gamkrelidze The exponential representation of flows and the chronological calculus, *Math. USSR Sb.* **106** (1978), 467–532.
- [2] H. Hermes, Lie Algebras of Vector Fields and Local Approximation of Attainable Sets, *SIAM J. Control Optim.* **16** (1978), 715–727.
- [3] M. Krastanov, M. Nikolova, A necessary condition for small-time local controllability *Automatica*, **15**, (online 2020), Volume 124 (2021), 109258.
- [4] M. Krastanov, M. Nikolova, On the “good” Lie brackets related to a polynomial system, *AIMS*, Volume 10, Issue 12 (2025).
- [5] A. Krener, The high order maximal principle and its applications to singular extremals, *SIAM J. Control Optim.* **15** (1977), 256–293.
- [6] J-B. Pomet, L. Giraldi, P. Lissy, Cl. Moreau, Necessary conditions for local controllability of a particular class of systems with two scalar controls, *ESAIM:COCM* **30** (2024).

- [7] Sussmann, H. J., A general theorem on local controllability, *SIAM J. Control and Optimization*, **25**, 158–193, (1987).